

# A parametric study of convective heat transfer on LNG tank by numerical simulations

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## Abstract

A computer code was developed, permitting to predict the phenomenon of the LNG thermal stratification, and to describe with precision its behavior at conditions close to the storage real conditions. The proposed method is based on the ADI method.

The aim of this study is to carry out a parameter setting of the principal physical properties intervening in the natural convection equations, namely the Rayleigh number characterizing heat exchange, Prandtl number which characterizes the relative importance of the heating and viscous effects, and the aspect ratio describing the tank geometry.

The obtained results show that for small values of Rayleigh number a flow exists, and results in weak variations of isotherms. The increase of Rayleigh number causes the increase of the maximum velocity no matter the dimension of the tank. Nevertheless, this increase is more significant in the case of high aspect ratio (less broad tank). We note that the disturbance reaches the center of the tank more quickly when this one is less broad. For the fluids of great Prandtl number (Ethylene glycol for example,  $Pr = 92.5$ ), the thermal time is larger than viscous time and the heat diffusion processes control the fluid motion.

*Keywords: natural convection, ADI, stratification, numerical simulation, sensitivity.*

## Nomenclature

$a$	Thermal diffusivity, $m^2/s$
$D$	Diameter of the tank, $m$
$G$	Acceleration of gravity, $m/s^2$
$H$	Height of the tank, $m$
$P$	Pressure, $pa$
$r, z$	dimensionless cylindrical coordinates
$u, v$	dimensionless velocities in $r, z$ directions

### Greek Symbols

$\beta$	Thermal expansion coefficient, $K^{-1}$
$\Delta T_0$	Temperature difference, $K$
$\theta$	dimensionless temperature
$\nu$	kinematic viscosity, $m^2/s$
$\tau$	dimensionless time
$\psi$	stream function
$\Omega$	dimensionless modified vorticity

### Non-dimensional Numbers

$Pr$	Prandtl number, $[\nu/a]$
$Ra$	Rayleigh number, $[g\beta\Delta T_0 H^3/\nu a]$

## 1. Introduction

To stock liquefied gases under atmospheric pressure, gas temperature must be lowered to their boiling point by an appropriated refrigeration system. The storage units must include some efficient thermal insulation systems in order to avoid the warming of cooled gas.

The storage tanks are characterized by their geometrical forms, their dimensions, the pressure and the temperature of the stored product which is related to its pressure by the saturated steam law.

It is obvious that as the insulating materials of the tank are of bad quality or lose their efficiency (ageing, cracking, etc), gains of heat through the walls of the tank cause a thermal stratification of the LNG.

The study of the natural convection in closed cavities caused many works, considering the new requirements as regards knowledge of dynamic and thermal properties of the industrial fluids.

Several works were the subject of studies on natural convection in cylindrical and rectangular cavities [1-5]. The numerical methods used to solve the equations are founded on the finite volume method [1-2], or the finite differences method the [3-5].

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In the study that we led, a computer code was developed, in order to predict the stratification phenomenon, and to describe with precision the fluid behavior under close-to-real storage conditions [6,7]. The proposed method suggested is based on the equations discretization by ADI method. It allows the resolution of the transient phenomenon using stable and convergent discretization diagrams.

The objective of the study is to make a parameter setting of the main physical properties intervening in equations of the natural convection problem, namely the Rayleigh number characterizing heat exchange, Prandtl number which characterizes the relative importance of the heating and viscous effects, and the aspect ratio describing the tank geometry. It consists in varying a parameter while fixing the two others and to examine the influence on temperature and velocity profiles.

## 2. Mathematical study

The natural convection is studied in two-dimension cylindrical tank, containing an incompressible fluid with constants physical properties (except for the density in the momentum equations). A uniform lateral heat flux causes the evaporation of a quantity of fluid (liquefied gas) at the upper free surface. The thermal entries at the bottom are neglected.

When the movements of the fluid are due to differences in temperatures, where the flow is produced by the fact that the variations in temperature involve variations of density, and under the effect of the gravity forces, densest layers go down with regards of the less dense layers.

The dimensionless conservation equations are:

$$\frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} = \frac{1}{Ra} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right) - \frac{\partial P}{\partial r} \quad (1)$$

$$\frac{\partial v}{\partial \tau} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} = \frac{1}{Ra} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right) - \frac{\partial P}{\partial z} \quad (2)$$

$$\frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial r} + v \frac{\partial \theta}{\partial z} = \frac{1}{\sqrt{RaPr}} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (3)$$

$$\frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial r} + v \frac{\partial \theta}{\partial z} = \frac{1}{\sqrt{RaPr}} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (4)$$

By introducing the vorticity and stream function concepts:

$$\Omega = r \left( \frac{\partial u}{\partial z} - \frac{\partial v}{\partial r} \right) \quad (5)$$

$$u = \frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial r} \quad (6)$$

One thus has:

$$\Omega = -\frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} \quad (7)$$

This equation can be regarded as a particular case (permanent) of the following general case (variable):

$$\frac{\partial \psi}{\partial \tau} = -\Omega - \frac{1}{r} \frac{\partial \psi}{\partial r} + \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} \right) \quad (8)$$

If one derives the Eq. 2 with regard to z and the Eq.3 with regard to r, while using continuity equation, the subtraction of the 2 equations obtained gives us:

Initial and boundary conditions are given in dimensionless forms:

- At the initial time  $\tau = 0$ :  $\psi = 0, \Omega = 0, \theta = 0$

- At the centerline axis  $r = 0$ :

$$\psi = \frac{\partial \psi}{\partial z} = 0, \quad \Omega = 0, \quad \frac{\partial \theta}{\partial r} = 0$$

- On the lateral wall  $r = r_{\max} = D/2$  H:

$$\psi = \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial r} = 0, \quad \Omega = \frac{\partial^2 \psi}{\partial r^2}, \quad \frac{\partial \theta}{\partial r} = 1$$

- At the bottom  $z = 0$ :

$$\psi = \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial r} = 0, \quad \Omega = \frac{\partial^2 \psi}{\partial z^2}, \quad \frac{\partial \theta}{\partial z} = 0$$

- On the upper free surface  $z = 1$ :

$$\psi = \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial r} = 0, \quad \Omega = \frac{\partial^2 \psi}{\partial z^2}, \quad \frac{\partial \theta}{\partial z} = -4 \frac{H}{D}$$

## 3. Numerical simulation

The energy and vorticity equations are solved by the ADI method -Alternating Direction Implicit- [8,9]. The tridiagonal linear systems obtained are solved by the Thomas algorithm.

The iterative method of gauss is used to solve the equation of the stream function. In this case, the calculation stops when the maximum difference between iterations becomes lower than  $10^{-5}$ .

The simulations allow describing the variable regime before reaching the steady state.

$$\text{The convergence criterion is } \frac{\sum_i \sum_j |F_{i,j}^{n+1} - F_{i,j}^n|}{\sum_i \sum_j |F_{i,j}^{n+1}|},$$

where F is the temperature or the vorticity. Variations by less than  $10^{-7}$  are adopted for convergence at each time step.

## 4. Results and discussion

The study of the natural convection problem in a cylindrical tank is related to three parameters: the aspect ratio H/D, the Prandtl number Pr and the Rayleigh number Ra.

Numerical tests were necessary to optimize the time and the precision of calculations. Thus, a regular mesh of

21x21 was found sufficient to simulate the distributions of flow and temperature in a cylindrical enclosure with  $0.125 \leq H/D \leq 0.5$ . These aspect ratios are lower than 1 because they actually correspond to storage tanks which in general are partially filled.

#### 4.1. Effect of Rayleigh number

Figures 1 and 2 show the effect of Rayleigh number for constant aspect ratio. It is noticed that even for small values of the number Rayleigh (when heat is mainly exchanged by conduction) a flow exists, and results in weak variations of isotherms. These variations become increasingly important as  $Ra$  increases. One notice that

the maximum value of temperature  $T_{\max}$  corresponds to small values of  $Ra$  (Figures 3,4 and 5). It is also noted that the minimal temperature decrease with the increase of Rayleigh number.

Figures 6 et 7 show the influence of the Rayleigh number on maximum velocity. One notices that the increase of Rayleigh led to an increase of maximum velocity either the tank dimension. Nevertheless, this increase is more significant in the case of high aspect ratio (less broad tank).

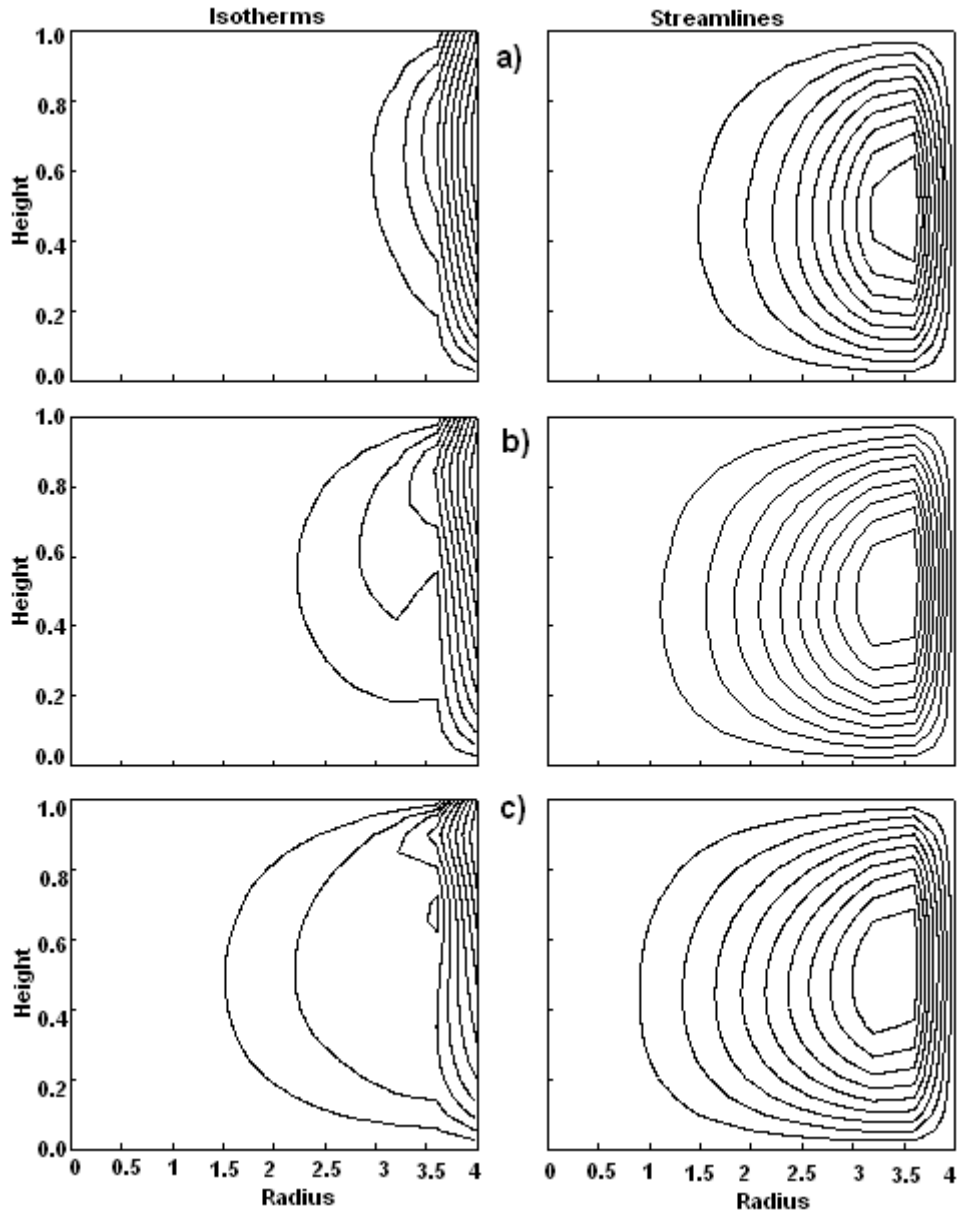


Fig .1 Effect of Rayleigh number on isotherms and streamlines for  $H/D=0.125$

(a)  $Ra = 1E03$  ,  $T_{\max} = 0.41$  ,  $T_{\min} = 0.039$  ,  $\psi_{\max} = 0.18$

(b)  $Ra = 1E04$  ,  $T_{\max} = 0.32$  ,  $T_{\min} = 0.032$  ,  $\psi_{\max} = 0.24$

(c)  $Ra = 5E04$  ,  $T_{\max} = 0.28$  ,  $T_{\min} = 0.027$  ,  $\psi_{\max} = 0.28$

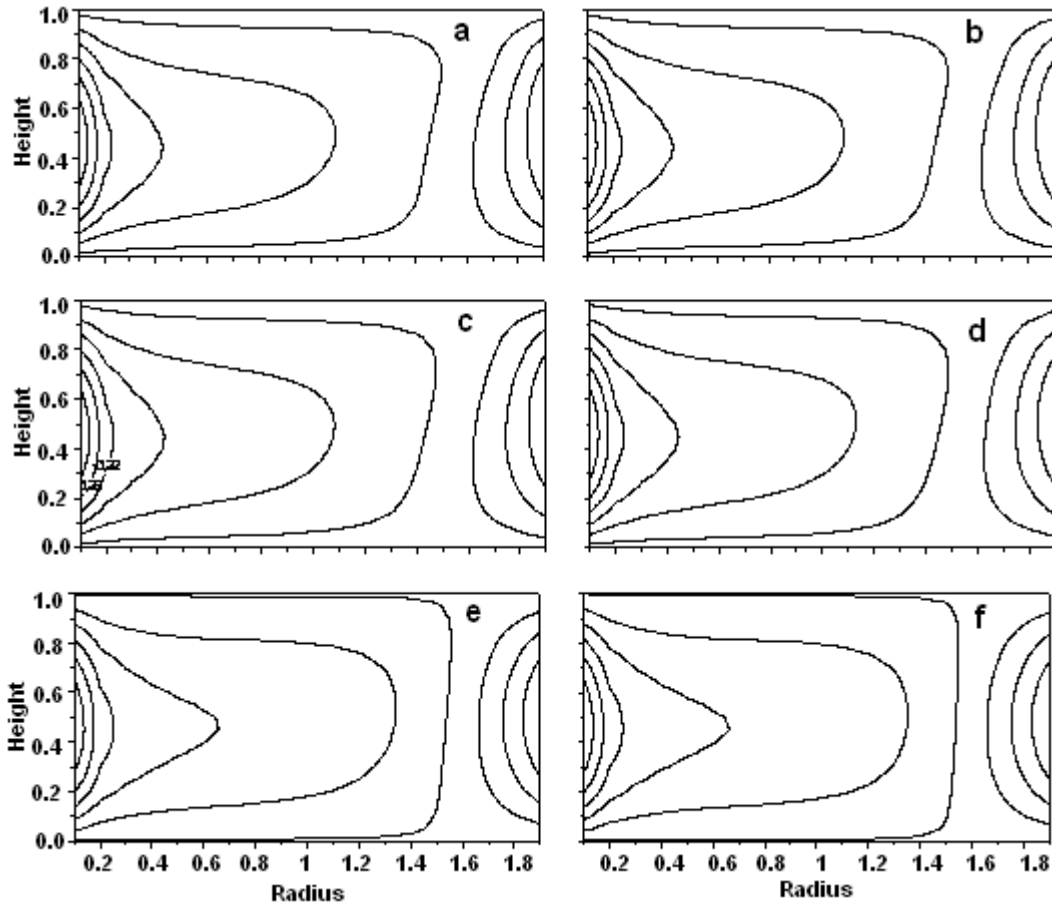


Fig.2. Effect of Rayleigh number on flow field in steady state for  $H/D=0.25$ ,  $Pr=2.2$   
(a)  $Ra = 2.E4$  ; (b)  $Ra=1.5.E4$  ; (c)  $Ra=1.E4$  ; (d)  $Ra=5.E3$ , (e)  $Ra=1.E3$ , (f)  $Ra=200$ .

#### 4.2. Effect of tank geometry

Figure 8 shows the effect of the tank aspect ratio  $H/D$  for  $Ra = 10^3$ . In this case, we note that the perturbation reaches more quickly the center of the tank when this one is less broad. Otherwise, profiles of temperature prove that the cold zone spreads progressively on the surface of the cylinder as the aspect ratio increases. Contrary to the case of the influence of  $Ra$ , we noticed that the maximum temperature increases with the increase of the aspect ratio  $H/D$  (Figure 9).

#### 4.3. Effect of Prandtl number

Prandtl number characterizes the competition of viscous and thermal diffusion effects and, therefore, is an influential factor in thermal-fluid flows. One notes thus that for the fluids of great Prandtl number (Ethylene glycol for example,  $Pr = 92.5$ ), thermal time is larger than viscous time and the heat diffusion processes control the movement of the fluid. For the low values of Prandtl number (liquid Hydrogen for example,  $Pr = 1.29$ ), the thermal effects are reduced and the fluid behavior is essentially streamlined.

Numerical experiments are carried out with different fluids such as  $1.29 \leq Pr \leq 92.5$ .

Figures 10 and 11 show that the maximum vertical velocity increases with the reduction of the Prandtl number, what encourages the thermal stratification.

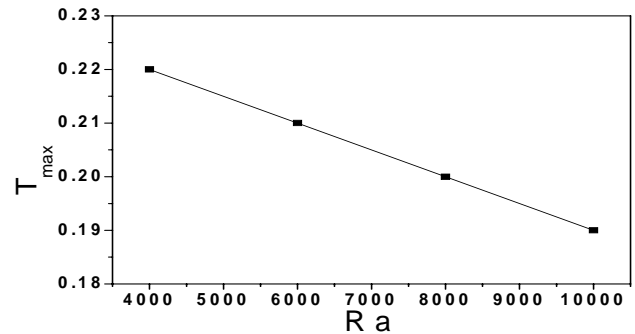


Fig. 3. Evolution of maximal temperature versus  $Ra$  in steady state for  $H/D = 0.5$

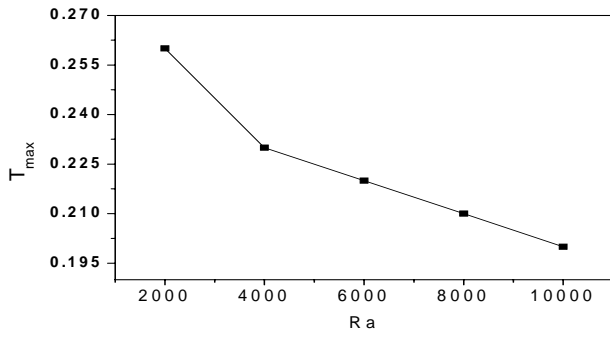


Fig. 4. Evolution of maximal temperature versus Ra in steady state for  $H/D = 0.25$

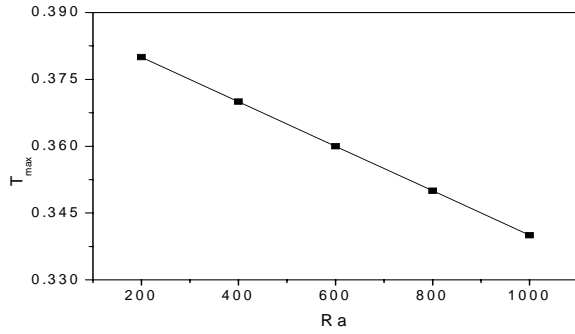


Fig. 5. Evolution of maximal temperature versus Ra in steady state for  $H/D = 0.125$

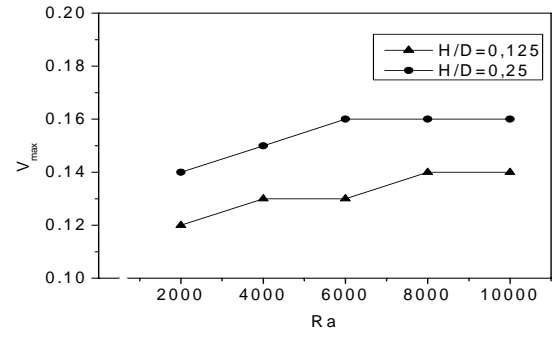


Fig. 6. Evolution of maximal vertical velocity versus Ra in steady state for  $H/D=0.125$  and  $H/D=0.25$

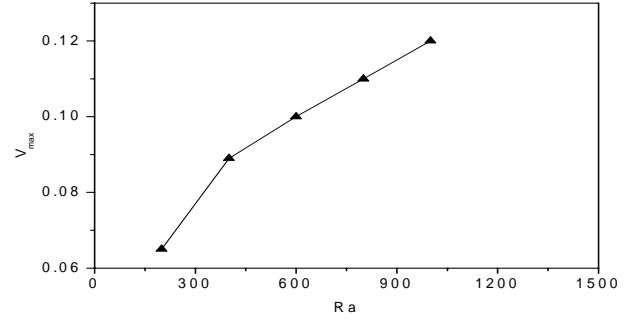


Fig. 7. Evolution of maximal vertical velocity versus Ra in steady state for  $H/D=0.5$

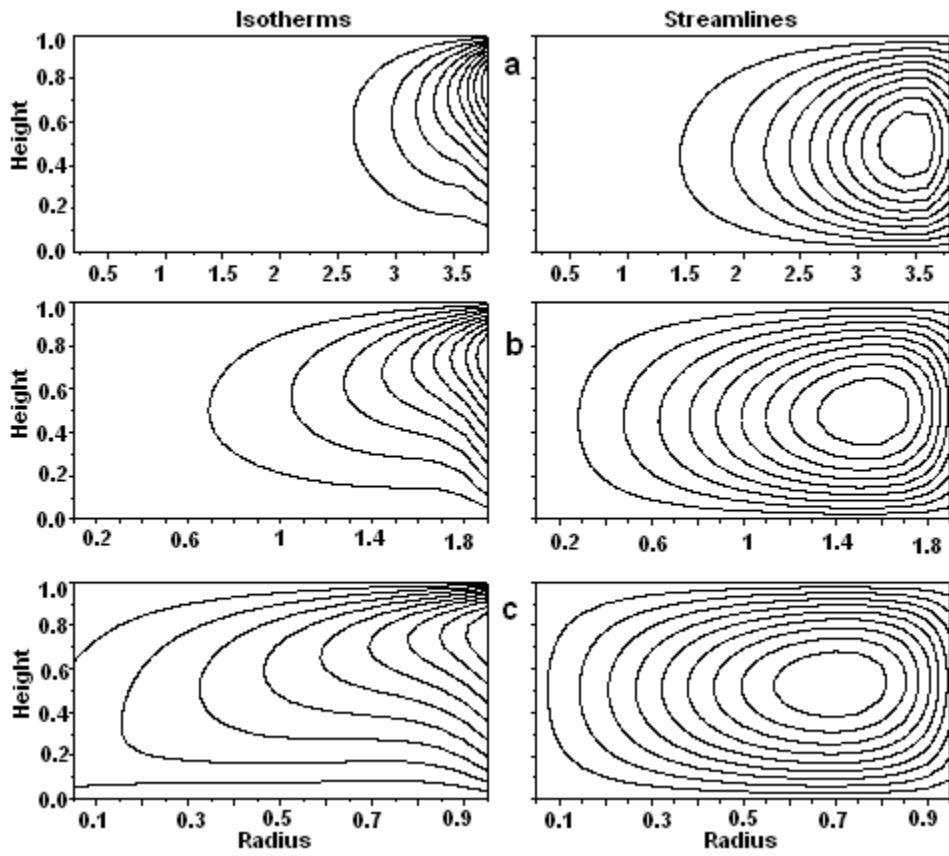
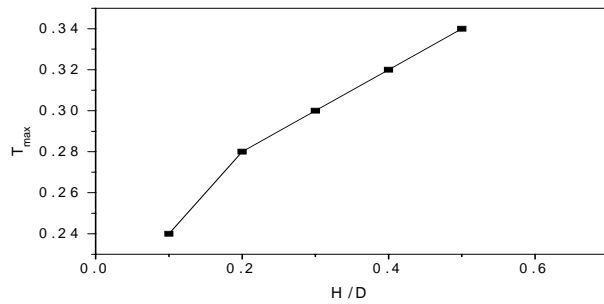
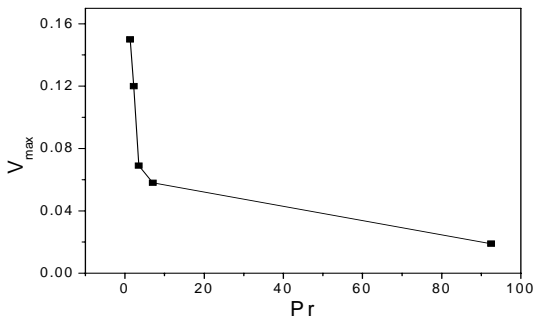


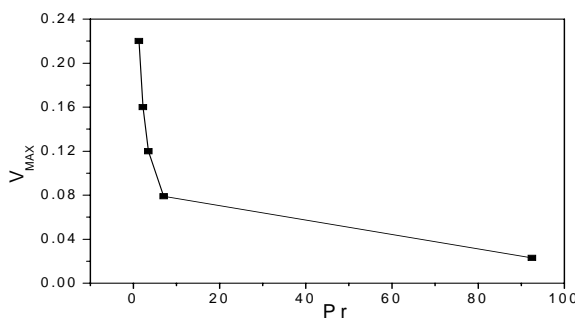
Fig. 8. Effect of tank geometry for  $Ra=1.E3$   
(a)  $H/D=0.125$  ; (b)  $H/D= 0.25$  ; (c)  $H/D = 0.5$



**Fig. 9. Evolution of maximal temperature versus H/D in steady state for Ra=1E3**



**Fig.10. Evolution of maximal vertical velocity versus Pr in steady state for H/D=0.125 and Ra=2E03**



**Fig.11. Evolution of maximal vertical velocity versus Pr in steady state for H/D=0.25 and Ra=1E04**

## 5. Conclusion

To study the effect of the principal parameters (Ra, Pr, H/D) on fluid flow in a cylindrical storage tank, a numerical simulation based on ADI method is presented. The results show that for small values of the Rayleigh number (where heat is exchanged mainly by conduction) flow exists, and is reflected by small changes in isotherms. These variations are becoming increasingly important as Ra increases. On the other hand, the maximum temperature  $T_{max}$  corresponds to small values of Ra. We note also that the increase in Rayleigh fact increase the maximum velocity regardless of the tank dimension. However, this increase is more significant in the case of high aspect ratio (tank less broad). We note that the perturbation reaches more quickly the center of the tank when it is less broad. For the fluids of great number of Prandtl (Ethylene glycol for example,  $Pr =$

92.5), thermal time is larger than viscous time and the processes of heat diffusion control the movement of the fluid. For low values of Prandtl number (Liquid hydrogen for example,  $Pr = 1.29$ ), thermal effects are reduced and the fluid behavior is essentially hydrodynamic.

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